Journal of Structural Geology, Vol. 4, No. 1, pp. 63 to 67, 1982.
Printed in Great Britain

0191-8141/82/010063-05 $03.00/0

The problem of shearing along axial plane foliations

S. K. GHOSH

Department of Geological Sciences, Jadavpur University, Calcutta 700 032, India

(*Received* 15 *December* 1980; *accepted in revised form* 3 *August* 1981)

Abstract—The structural significance of axial plane foliations cannot be understood unless we make a distinction between rotation of the material plane foliation and rotation of the geometrically defined XY-plane of the strain ellipsoid. If the foliation rotates as a material plane at any stage of deformation, then its final orientation will be different from that of the XY-plane. It is suggested that reorientation of foliation takes place by some combination of the formation of foliation (e.g. recrystallization) along the XY-plane and passive rotation of the material plane foliation in the same continuous deformation. The deviation between the foliation and the XY-plane is then much less than 5 degrees. However, because of this deviation, a considerable amount of shear strain may develop along the foliation. The analysis, thus, explains how a foliation can be approximately parallel to the XY-plane and yet be a plane of shearing.

INTRODUCTION

THE AXIAL-PLANE foliation is usually regarded as a structure which develops essentially parallel to the XY-plane of the strain ellipsoid. Field evidence clearly indicates that the foliation is approximately perpendicular to the direction of maximum shortening. However, in certain foliated rocks, cross-cutting markers are found to be offset along discrete foliation surfaces (Hills 1945, Naha & Ray 1972). As Hobbs *et al.* (1976, p. 237) point out: "We are thus faced with the problem of explaining how a foliation can be a plane parallel to which shearing displacements have occurred, and yet be parallel, or approximately parallel, to a principal plane of the strain ellipsoid (a principal plane of strain is a plane of no shear strain)". The following discussion is an attempt to analyse this problem for the general case of rotational deformation.

GENERAL CONSIDERATIONS

The argument is often given that the foliation parallel to the XY-plane cannot have any shearing on it because, by definition, the XY-plane is a plane of zero shear strain. Apparently, the argument sounds sufficiently straightforward. However, when we go deeper into the problem, we find that this argument is rather confusing.

The orientations of the principal axes of strain are defined as the initial orientation of that set of mutually perpendicular material lines which remain mutually perpendicular at the close of deformation. The angle between their final and initial positions is defined as finite rotation (W). The lines do not remain mutually perpendicular in any intermediate stage of rotational deformation; hence there must be shear strains corresponding to these lines throughout the entire course of deformation. The principal axes and their final positions are so defined that the sum of the shear strains becomes zero at the end of the period of deformation. It is obvious, then, that at some stage of progressive deformation, the sense of shear corresponding to these lines is reversed, so that, at the close of deformation, the absolute value of positive shear exactly balances the absolute value of negative shear. At the stage of deformation when reversal in the sense of shear takes place, the orientations of the lines do not coincide with those of the principal axes of infinitesimal strain. In other words, this orientation does not bisect the angle between the initial and final orientations of the principal axes of strain. This is shown below by an analysis of the different stages of simple shear.

In Fig. 1, a unit square with sides OA and OB is shown to be deformed by simple shear (γ) in the x-direction into a parallelogram with sides OC and OD. The initial line OA and the corresponding deformed line OC make with the x coordinate axis the angles α and α', respectively. The right angle AOB has changed to the angle COD or $(\frac{\pi}{2} - \psi')$, where ψ' is the shear angle. $\gamma'(= \tan \psi')$ is the shear strain corresponding to the lines OC and OD in the deformed state. Then:

$$\gamma' = \frac{\gamma^2}{2} \cdot \sin 2\alpha + \gamma \cos 2\alpha \qquad (1)$$

(Nadai 1950, eqns. 13–44, p. 147).

It should be noted that γ' is the shear strain corresponding to a pair of lines whose initial orientations were α and $(\frac{\pi}{2} + \alpha)$, while γ is the amount of simple shear corresponding to the coordinate axes x and y. If α and $(\frac{\pi}{2} + \alpha)$ are regarded as the initial and the final positions of the principal axes of strain, then at the close of deformation γ' should be zero. From (1), this condition is given as:

$$\frac{\gamma}{2} \sin 2\alpha + \cos 2\alpha = 0 \qquad (2)$$

or:

$$\gamma = \frac{-2}{\tan 2\alpha}. \qquad (3)$$

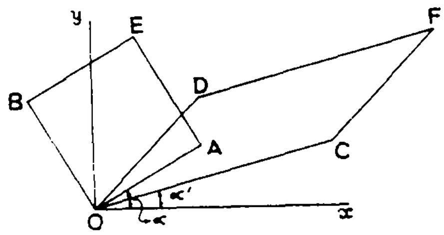

Fig. 1. (a) Deformation of the square OAEB into the parallelogram OCFD by simple shear. The direction of simple shear movement is along the x-axis. OA and OB are the initial orientations of two lines at angles of α and $\frac{\pi}{2} + \alpha$, respectively. The angle between OC and OD is $(\frac{\pi}{2} - \psi')$, so that $\tan \psi'$ is the shear strain corresponding to these two lines.

This is the value of simple shear in the x-direction at the close of deformation when the material lines are once again mutually perpendicular.

Next, we seek to find the stage of simple shear (say, γ_A) at which sense of shear along the material lines is reversed. At this stage γ' itself will not change its sign; its value will remain momentarily stationary (point A in Fig. 2). This condition is:

$$\frac{d\gamma'}{d\gamma} = 0. \tag{4}$$

By substituting the expression for γ' from (1) into (4), we find that

$$\gamma_A = \frac{-1}{\tan 2\alpha}. \tag{5}$$

This is the value of simple shear at which the sense of shear along the material lines is reversed. A comparison of (5) with (3) shows that

$$\gamma_A = \frac{1}{2}\gamma. \tag{6}$$

Thus, the reversal in the sense of shear takes place when the deformation has progressed exactly halfway. This first half of deformation rotates the material line through an angle of more than $\frac{1}{2}$ W, W being the angle between the final and initial positions of the principal axes.

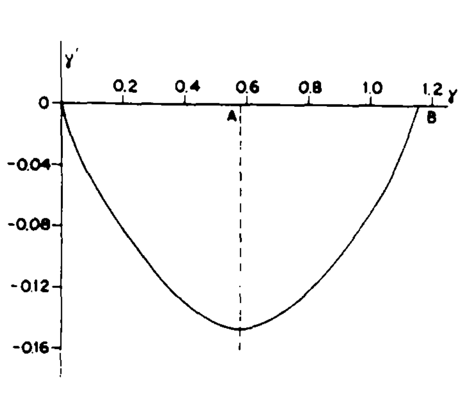

Fig. 2. Variation of shear strain γ' corresponding to two lines in simple shear. One of the lines was initially at an angle of 60° with the x-axis. γ is the amount of simple shear. The lines were mutually perpendicular at the beginning and at the end (point B) of the deformation. The maximum of the absolute value of γ' is obtained when the deformation has progressed exactly halfway (point A). The sense of shear is reversed at this point.

From the analysis given above, it is clear that γ' can vanish only when the material lines rotate through the entire angle W. For our purpose, a point of crucial importance is that *the foliation could never be parallel to the initial orientation of the principal axes of strain.* If the foliation finishes up parallel to the XY-plane of the strain ellipsoid, then its initial orientation must lie somewhere between the orientation of the XY-plane of the finite strain ellipsoid and the orientation of the XY-plane of the infinitesimal strain ellipsoid. Thus, the rotation of the foliation cannot be more than $\frac{1}{2}$ W. In contrast, a material line which was once parallel to the initial orientation of the principal axes of strain must have rotated through the entire angle of W at the close of deformation. Indeed, if we assume that the foliation is all the time parallel to the changing orientations of the XY-plane of the strain ellipsoid, then, we cannot meaningfully raise the question about the presence or absence of shear strain along it because, in that case, the foliation would not be defined by the same material plane in successive instants of time.

If reorientation of a foliation takes place by rotation of rigid grains in a matrix, the directional change in their preferred orientation may be different from that of a passive (Turner & Weiss 1963, p. 391) material plane. The equations for rotation of isolated rigid ellipsoidal grains embedded in a deformable matrix (Jeffery 1922, Ghosh & Ramberg 1976, eqns. 11, 12 and 13, Ghosh 1977, fig. 13) are quite different from that of the passive rotation of markers. However, if the ratios of the longest and shortest axes of the grains are large (i.e. more than 5), the values of their rigid rotations come very close to that of the passive rotation of a corresponding marker (Ghosh & Ramberg 1976, p. 23). If a large number of rigid grains are closely spaced within a matrix, the corresponding equations of rotation are likely to be different from those of a single isolated grain. There is, however, no reason to believe that the longest axes of the grains will continuously trace out the orientations of the X-axes of the bulk strain ellipse. Preliminary simple-shear tests carried out in the Department of Geological Sciences of the Jadavpur University suggest that, when strongly elongate rigid inclusions are closely packed within a viscous matrix, the directional change of their preferred orientation is virtually indistinguishable from that of a corresponding passive marker plane. In these tests, the intermediate axes of the inclusions were parallel to the axis of simple shear.

But, after all, what do we really mean when we say that the axial plane foliation is parallel to the XY-plane? In the general case of rotational deformation, the orientation of the XY-plane continuously changes. A material plane, which was once parallel to the XY-plane, can never again coincide with it at any other stage of progressive deformation. Do we, then, mean that the foliation never behaves as a material surface during progressive deformation. In other words, do we mean that the elongate grains such as the flakes of mica, never rotate as material entities for any instant of time, but crystallize anew in an infinitesimally different orientation at successive instants of progressive deformation? Such wholesale instantaneous recrystallization is unlikely, especially when the foliation is marked by coarsely crystalline grains. Certainly, the alternative, that reorientation of foliation is achieved by some combination of crystallization along the XY-plane and passive rotation of the material plane of foliation, is much more acceptable (Bayly 1974). We can think about different kinds of such combinations. For instance, the foliation may acquire a well-defined character and be parallel to the XY-plane at a certain stage of deformation; with continued deformation, the foliation may rotate as a material plane. Alternatively, development of the foliation along the XY-plane and its rotation as a material plane may take place in alternate steps. In either case, rotation of the material plane of foliation will be accompanied by a shear strain along it. In the next section an attempt will be made to determine the maximum possible angular divergence between the foliation and the XY-plane and to determine the resulting shear strain on it. It should be noted that the rotation of the foliation need not be entirely post-crystalline; the foliation may continue to be regenerated parallel to the rotating material plane by syntectonic mimetic crystallization.

DEVELOPMENT OF FOLIATION ALONG THE XY-PLANE AND ITS SUBSEQUENT ROTATION AS A MATERIAL PLANE

If the foliation is initiated parallel to the XY-plane and is subsequently rotated as a material plane in the same continuous deformation, the maximum possible angular divergence between the XY-plane and the foliation will always be small. Ghosh (1975, p. 207) showed that in simple shear deformation, this divergence will always be less than 5.15 degrees. It will be shown here that, if the deformation is by combined pure shear and simple shear, then this angular divergence will be still smaller (Fig. 3).

For combined pure- and simple- shears, we choose the following particle-path equation:

$$x = \exp(\varepsilon_x)x_0 + \frac{1}{s_r}(\sinh \varepsilon_x)y_0 \qquad (7)$$

$$y = \exp(-\varepsilon_x)y_0.$$

where ε_x is the natural strain for pure shear and s_r is the ratio $(\dot{\varepsilon}_x/\dot{\gamma})$ of the rates of pure shear and simple shear (Ramberg 1975, Ghosh & Ramberg 1977, p. 23). It may be noted that $\varepsilon_x = \gamma s_r$. Then, orientations of the principal axes of the finite strain ellipse are given by the equation:

$$\tan 2\theta' = \frac{\frac{1}{s_r}\{1 - \exp(-2\gamma s_r)\}}{\left\{\frac{1}{s_r}\sinh(\gamma s_r)\right\}^2 + \{\exp(2\gamma s_r) - \exp(-2\gamma s_r)\}}. \qquad (8)$$

If θ_0 and θ are the initial and final positions of a marker line on the XY-plane, we obtain the following relation from (7):

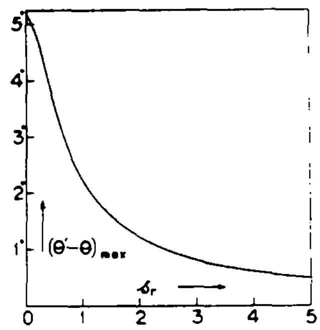

Fig. 3. Variation of $(\theta' - \theta)_{max}$ with s_r, where $(\theta' - \theta)_{max}$ is the maximum possible angle between the foliation and the XY-plane in combined pure and simple shears. Initially, θ' and θ coincided with the major axis of the infinitesimal strain ellipsoid for each value of s_r. With progressive deformation the foliation rotated as a material plane. Note that the maximum value of $(\theta' - \theta)$ is obtained when $s_r = 0$. This value is 5.15 degrees. $(\theta' - \theta)$ decreases as s_r increases.

$$\tan\theta = \frac{\tan\theta_0 \cdot \exp(-\gamma s_r)}{\exp(\gamma s_r) + \frac{1}{s_r}\sinh(\gamma s_r)\tan\theta_0}. \qquad (9)$$

In order to determine the maximum possible difference in angle between the XY-plane and the foliation, it will be assumed that the schistosity had initially developed parallel to the XY-plane of the infinitesimal strain ellipsoid. For combined pure shear and simple shear, the orientation (θ_0) of the XY-plane of the infinitesimal strain ellipsoid is given by the following equation:

$$\theta_0 = \frac{1}{2}\tan^{-1}\left(\frac{1}{2s_r}\right). \qquad (10)$$

If, from this initial position, the foliation rotates as a marker plane, then its final orientation, θ, can be obtained from (9) and (10):

$$\tan\theta = \frac{\exp(-\gamma s_r)\cdot\tan\left\{\frac{1}{2}\tan^{-1}\left(\frac{1}{2s_r}\right)\right\}}{\exp(\gamma s_r) + \frac{1}{s_r}\sinh(\gamma s_r)\cdot\tan\left\{\frac{1}{2}\tan^{-1}\left(\frac{1}{2s_r}\right)\right\}} \qquad (11)$$

$(\theta' - \theta)$ can, then, be calculated from (8) and (11). The values of $(\theta' - \theta)$ with progressive deformation, always shows a maximum (Fig. 3), unless of course, the deformation is by pure shear, in which case θ' and θ coincide. Numerical calculations show that the maximum value of $(\theta' - \theta)$ is obtained when $s_r = 0$ (Fig. 3). As s_r increases, the maximum angular divergence decreases (Fig. 3). Figure 4 shows the variation of $(\theta' - \theta)$ for $s_r = 1$. The maximum angular divergence in this case is about 2.22 degrees. If the foliation had developed after a certain amount of initial deformation, the maximum value of $(\theta' - \theta)$ would be less than 2.22 degrees.

Thus, according to this model, it is expected that the angle between the foliation and the XY-plane will be very small. In most cases the angle will be too small to be detected in the field. The foliation would then appear to be

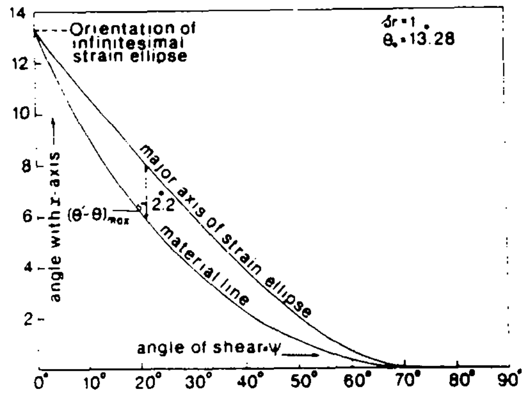

Fig. 4. Change in orientation of the X-axis and a material line when $s_r = 1$. It is assumed that the material line coincided with the X-axis of the infinitesimal strain ellipse. The maximum possible divergence between the two lines is 2.2 degrees. $\gamma (= \tan \psi)$ represents the simple shear part of the deformation.

approximately perpendicular to the direction of maximum shortening. However, this small angular divergence is of considerable theoretical importance, since a fairly large amount of shear strain can now develop along the foliation. For instance, if $s_r = 1$ and $\gamma = 1$, the angle $(\theta' - \theta)$ as calculated from (8) and (11), shows a small value of 1.2 degrees. The shear strain γ' on the foliation will, however, be as large as 1.5.

It has been assumed in the foregoing discussion that the foliation is initiated parallel to the XY-plane and is subsequently rotated in the same continuous deformation. There is, however, no criterion available to prove (or, for that matter, disprove) that the foliation is initiated along the XY-plane. The assumption has been made, because, as yet, it gives the most general explanation for the development of shear strain along the foliation and for the approximate parallelism of the final orientations of the foliation and the XY-plane. In particular, this theoretical model does not require an assumption of large shortening; the angular divergence always remains small (i.e. less than 5.15 degrees for simple shear and still smaller for other types of deformation), whatever be the shortening along the Z-axis. Hobbs *et al.* (1976, p. 242) have considered the possibility that the foliation is initiated parallel to a plane of high shear strain. The angle between the XY-plane and the planes of maximum shear strain is

$$\tan^{-1}\left(\frac{\lambda_3}{\lambda_1}\right)^{1/2}$$

(Jaeger 1964, p. 38, Ramsay 1967, p. 153), where λ_1 and λ_3 are the quadratic elongations parallel to X and Z, respectively. Hence, it is possible to have a very small angle if (λ_3/λ_1) is sufficiently small. For instance, for plane strain (i.e. when $\lambda_2 = 1$), this model predicts that the angle between the XY-plane and the lines of maximum shear strain will be less than 10 degrees when the shortening along the λ_3 − axis is in excess of 60 per cent (Hobbs *et al.* 1976, p. 242). It may be pointed out that, for the same value of shortening, the angle would be larger if $\lambda_2 > 1$; the maximum angular divergence would be obtained when $\lambda_2 = \lambda_1$. Thus, for a 60 per cent shortening along the λ_3 − axis, the angular divergence would be 14.2° when $\lambda_2 = \lambda_1$. For different values of shortening along the λ_3 − axis, Table 1 compares the values of angular divergence for the two cases, namely when $\lambda_2 = 1$ and when $\lambda_2 = \lambda_1$. The Table shows that in the present context, this model would be relevant when the shortening (including the local shortening at the fold-hinge and limbs) is rather high.

CONCLUSIONS

That the axial-plane foliation may deviate from the XY-plane is certainly not a new idea. A passive early foliation may rotate as a material plane in a second deformation. As pointed out by Ramsay (1967) and by Bayly (1974), its final orientation may not then coincide with the XY-plane for the combined deformation. Depending on the nature of the two deformations, the deviation may be large or small. In this analysis, however, we are concerned with a single continuous rotational deformation. It is common knowledge (Bayly 1974, Dzis 1976, Elliott 1972, Ghosh & Sengupta 1973, Ghosh 1975, Matthews *et al.* 1971, 1976, Owen 1973, Ramberg & Ghosh 1977, Williams 1976) that, if the foliation rotates as a material plane at any stage of deformation, then its final orientation will deviate from that of the XY-plane. In simple shear deformation, the deviation will always be less than 5.15 degrees (Ghosh 1975, p. 207). The preceding analysis has shown that, if the deformation is by combined pure- and simple- shears, then, the deviation will be still smaller. However, because of this deviation, a considerable amount of shear strain may be generated along the foliation. This analysis, therefore, explains how a foliation can be approximately parallel to the XY-plane and yet be a plane of shearing. In addition, if s_r is sufficiently large, the foliation will make a very small angle with the direction of movement (x-axis) of simple shear part of the deformation. This would explain the development of a mylonitic foliation on which a considerable amount of shear has taken place and which occurs at very low angles with both the XY-plane and the thrust plane. Moreover, in rotational deformation, there must be a component of shearing stress along the foliation. It is likely that these shearing stresses are mainly responsible for the development of discrete surfaces of slip along the foliation (Dieterich 1969).

Table 1. Angle between XY-plane and lines of maximum shear strain

Percentage of shortening along λ_3 − axis	Angle (in degrees) between XY-plane and lines of maximum shear strain when $\lambda_2 = 1$	when $\lambda_2 = \lambda_1$
30	26.1	30.4
40	19.8	24.9
50	14.0	19.5
60	9.1	14.2
70	5.1	9.3

REFERENCES

Bayly, M. B. 1974. Cleavage not parallel to finite-strain ellipsoid's *XY*-plane: discussion. *Tectonophysics* **23**, 205–208.

Dieterich, J. H. 1969. Origin of cleavage in folded rocks. *Am. J. Sci.* **267**, 155–165.

Dzis, R. J. 1976. Finite and incremental deformations recorded by planar subfabrics in S-tectonites. *Tectonophysics* **36**, 367–393.

Elliott, D. 1972. Deformation paths in structural geology. *Bull. geol. Soc. Am.* **83**, 2621–2638.

Ghosh, S. K. 1975. Distortion of planar structures around rigid spherical bodies. *Tectonophysics* **28**, 185–208.

Ghosh, S. K. 1977. Drag patterns of planar structures around rigid inclusions. In *Energetics of Geologic Processes* (edited by Saxena, S. K. & Bhattacharji, S.) Springer-Verlag, New York, 94–120.

Ghosh, S. K. & Ramberg, H. 1976. Reorientation of inclusions by combination of pure shear and simple shear. *Tectonophysics* **34**, 1–70.

Ghosh, S. K. & Sengupta, S. 1973. Compression and simple shear of test models with rigid and deformable inclusions. *Tectonophysics* **17**, 133–178.

Hills, E. S. 1945. Examples of the interpretation of folding. *J. Geol.* **53**, 47–57.

Hobbs, B. E., Means, W. D. & Williams, P. F. 1976. *An Outline of Structural Geology*. John Wiley, New York.

Jeffery, G. B. 1922. The motion of ellipsoidal particles immersed in a viscous fluid. *Proc. R. Soc.* **A102**, 161–179.

Matthews, P. E., Bond, R. A. B. & van den Berg, J. J. 1971. Analysis and structural implications of a kinematic model of similar folding. *Tectonophysics* **12**, 129–154.

Matthews, P. E., Bond, R. A. B. & van den Berg, J. J. 1976. Strain analysis of symmetric folding using slaty cleavage. *Tectonophysics* **34**, 199–217.

Nadai, A. 1950. *Theory of Flow and Fracture of Solids*. McGraw-Hill, New York.

Naha, K. & Ray, S. K. 1972. Structural evolution of the Simla Klippe in the Lower Himalayas. *Geol. Rdsch.* **61**, 1050–1086.

Owen, W. H. 1973. Strain modification of angular density distribution. *Tectonophysics* **16**, 249–261.

Ramberg, H. 1975. Particle paths, displacement and progressive strain applicable to rocks. *Tectonophysics* **28**, 1–37.

Ramberg, H. & Ghosh, S. K. 1977. Rotation and strain of linear and planar structures in three-dimensional progressive deformation. *Tectonophysics* **40**, 309–337.

Ramsay, J. G. 1967. *Folding and Fracturing of Rocks*. McGraw-Hill, New York.

Turner, F. J. & Weiss, L. E. 1963. *Structural Analysis of Metamorphic Tectonites*. McGraw-Hill, New York, N.Y.

Williams, P. F. 1976. Relationships between axial-plane foliations and strain. *Tectonophysics* **30**, 181–196.